

U-Duality Multiplets and Non-perturbative Superstring States

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ABSTRACT

We employ an algebraic approach for unifying perturbative and non-perturbative superstring states on an equal footing, in the form of U-duality multiplets, at all excited string levels. In compactified type-IIA superstring theory we present evidence that the multiplet is labelled by two spaces, “index” space and “base” space, on which U acts without mixing them. Both spaces are non-perturbative extensions of similar spaces that label perturbative T-duality multiplets. Base space consists of all the central charges of the 11D SUSY algebra, while index space corresponds to representations of the maximal compact subgroup $K \subset U$. This structure predicts the quantum numbers of the non-perturbative states. We also discuss whether and how U -multiplets may coexist with 11-dimensional multiplets, that are associated with an additional non-perturbative 11D structure that seems to be lurking behind in the underlying theory.

I. INTRODUCTION

It has been conjectured that, in addition to T-duality and S-duality, string theory may possess an even larger duality, U-duality [1], that contains both T and S. There is plenty of evidence for the existence of S duality and circumstantial evidence for U-duality [1]- [10]. These ideas have provided a degree of re-unification of certain string theories that *à priori* seemed to be different. It now appears that there is a single underlying theory with many moduli fields, and that the various conformal string theories are different perturbative starting points from various corners of moduli space. However, each perturbative expansion misses non-perturbative aspects that may

have already been described by another perturbative expansion. Furthermore, beginning with 11D supergravity in the form of the low energy limit, there seems to be an 11 dimensional structure lurking behind the non-perturbative aspects of these string theories [2], [11]- [14]. Further study of duality is bound to reveal more non-perturbative properties of the underlying theory.

In this paper we investigate the multiplet structures for T and U-duality transformations and use it as a device for unifying the perturbative and non-perturbative states of the theory. This allows us to put all the states on the same footing. Our discussion sheds new light into the non-perturbative symmetry structure of the underlying theory and provides an algebraic tool for discovering the non-perturbative states.

We will mainly discuss the case of the 10-dimensional type-IIA superstring toroidally compactified on $R^d \times T^c$, that has a d -dimensional Minkowski spacetime with $c = 10 - d$ dimensions compactified on tori. The T -duality group is [15]

$$T = O(c, c; Z) \quad (1.1)$$

in all cases. The conjectured non-compact U -groups, their maximal compact subgroups $K \subset U$, and the maximal compact subgroup k of the T-group,

$$k = O(c) \times O(c) \quad (1.2)$$

are listed for various dimensions in the following table [1]. Note that since $T \subset U$ then $k \subset K$. It is understood that these groups are continuous in supergravity but only their discrete version can hold in string theory.

d/c	U	K	k
9/1	$SL(2) \otimes SO(1, 1)$	$U(1) \otimes Z_2$	$Z_2 \otimes Z_2$
8/2	$SL(3) \otimes SL(2)$	$SO(3) \otimes U(1)$	$U(1) \otimes U(1)$
7/3	$SL(5)$	$SO(5)$	$SO(4)$
6/4	$SO(5, 5)$	$SO(5) \otimes SO(5)$	$SO(4) \otimes SO(4)$
5/5	$E_{6(6)}$	$USp(8)$	$Sp(4) \otimes Sp(4)$
4/6	$E_{7(7)}$	$SU(8)$	$SU(4) \otimes SU(4)$
3/7	$E_{8(8)}$	$SO(16)$	$SO(7) \otimes SO(7)$

TABLE I. Duality groups and compact subgroups.

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T-duality was originally understood in toroidal backgrounds, but the scope of T-duality in string theory is much larger and it exists in more complicated curved backgrounds involving both compact and non-compact spacetimes [15], in particular in all gauged WZW models. T-duality transformations act on perturbative string states, and it can be verified that T-duality is valid order by order in string perturbation theory as well as in string field theory [16]. Target spaces related to each other by T-duality give the same physical results for T-invariant quantities such as the partition function. Different string target spaces are the analogs of different vacua, and T-duality are the analogs of large discrete gauge transformations that relate them.

The string states involved in the T-duality transformations are not all degenerate in mass. Therefore, T-duality must be regarded as the analog of a spontaneously broken symmetry, and the string states must come in complete multiplets despite the broken nature of the symmetry.

In this paper we will first clarify the nature of T-multiplets of excited string states and the properties of U-multiplets in low energy supergravity. By putting well known results [16] [17] into a suitable form we will emphasize the hints they provide for U-multiplets. Then we will use this information as boundary conditions to investigate the structure of non-perturbative U-multiplets, first in the supergravity sector ($l = 0$), including non-perturbative BPS saturates states, and then at all excited string levels l . In particular we will explicitly construct the U-multiplets for a number of low lying levels, up to level $l = 5$.

To avoid later confusion we give a definition of level l that applies to both perturbative and non-perturbative states. The mass of perturbative states includes contributions from Kaluza-Klein and winding numbers in addition to the string excitation level l . Thus, for perturbative states l is the excitation level of *oscillators*, not the mass. For the non-perturbative states we *define* the level l to be the same as the level of the perturbative states to which they are connected by duality transformations as explained below. Thus, l is a label of the entire U-multiplet. In the explicit structures that emerge, U-transformations, much like T-transformations, do not mix perturbative or non-perturbative states that belong to different levels (using the definition of level just given). This is an empirical observation that holds in our investigation up to level $l = 5$, and thus we assume that it holds at all levels. Our analysis critically depends on this assumption.

Our proposal is that a U-multiplet has the form

$$\Phi_{indices}^{(l)}(x^\mu, base) \quad (1.3)$$

where the indices and the base form complete multiplets under K and U respectively, for every spin. In the rest of the paper we will collect the evidence that leads to this structure and then use our proposal to make new statements about the non-perturbative theory. Both

the *index* space and the *base* space will include non-perturbative extensions. We will show how the full group U acts on this structure without mixing the index and base spaces at each level l .

Like in a T-multiplet, the states in a U-multiplet are not all mass-degenerate. But unlike a T-multiplet, a U-multiplet can contain perturbative states together with non-perturbative states (*minimal* case), or only non-perturbative ones (*non-minimal* case). In the minimal case, by knowing the structure of a U-multiplet as in (1.3) we can predict algebraically the quantum numbers of the non-perturbative states by extending the quantum numbers of the known perturbative T-multiplets that belong to the bigger U-multiplet. This completion provides the minimal set of non-perturbative states necessary for U-duality. There remains the non-minimal case, that includes the possibility of additional purely non-perturbative, complete U-duality multiplets. With additional input, our approach can provide information on some non-minimal U-multiplets, as we will discuss in section-V.

Therefore, our approach is an algebraic method that should complement the analytic methods of finding solutions for building up the non-perturbative spectrum of the theory. The non-perturbative spectrum that we find explicitly is one of the immediate outcomes of our approach.

In addition, our formulation hopefully provides hints for the non-perturbative formulation of the theory. In particular, a challenging question is whether U-multiplets can be made consistent with some deeper underlying structure. A possible candidate is an underlying 11-dimensional structure which has recently aroused much interest [2], [11]- [14]. We will find a certain amount of support for hidden 11-dimensions, but we will also raise some questions that require resolution.

II. T-MULTIPLETS

In this section we will show that the structure of perturbative T-multiplets at all levels l , is consistent with (1.3). In particular we want to show that both *base* and *index* spaces transform and that they do not mix with each other.

The bosonic string on $R^d \otimes T^c$ has the following *perturbative* states at oscillator level l

$$\phi_{indices}^{(l)}(x^\mu, base) \leftrightarrow (oscillators)^{(l)}|p^\mu, \vec{m}, \vec{n} > \quad (2.1)$$

where the *base* $\equiv (\vec{m}, \vec{n})$ are c -dimensional vectors on the Lorentzian lattice $\Gamma^{c,c}$ representing the Kaluza-Klein and winding numbers, while the *indices* are inherited from oscillators. For the superstring (in the Green-Schwarz formalism) the indices are also inherited from the Ramond vacuum, but without changing the overall structure. As mentioned above, these states are clearly not all degenerate in mass, however they are mixed with each other

under T-duality transformations. As we will see later we need to extend both the index and base spaces to construct U-multiplets.

It is well known that $T = O(c, c; Z)$ acts linearly on the $2c$ dimensional vector (\vec{m}, \vec{n}) [15]. However it also acts on the indices in definite representations. The action of T on the indices is an induced k -transformation that depends not only on *all* the parameters in T , but also on the background $c \times c$ matrices (G_{ij}, B_{ij}) that define the tori T^c . To see this we analyze the T-transformations in more detail. As is well known, $O(c, c; Z)$ acts linearly on the $2c$ dimensional vector (\vec{m}, \vec{n}) in such a way as to keep the following dot product invariant

$$(\vec{m}_1 \ \vec{n}_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \vec{m}_2 \\ \vec{n}_2 \end{pmatrix} = \vec{m}_1 \cdot \vec{n}_2 + \vec{n}_1 \cdot \vec{m}_2. \quad (2.2)$$

$O(c, c; Z)$ transformations are characterized by $2c \times 2c$ integer matrices of the form

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad M^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (2.3)$$

Our interest here is in understanding the action of the $O(c, c; Z)$ transformations on the oscillators, and hence on the indices of the fields in (1.3, 2.1). This can be extracted from a study of T -invariance of string field theory [16]: $O(c, c; Z)$ acts in such a way as to keep the left/right string excitation level numbers $N_L = \sum \alpha_{-n}^{Li} G_{ij} \alpha_n^{Lj}$ and $N_R = \sum \alpha_{-n}^{Ri} G_{ij} \alpha_n^{Rj}$ invariant. It does so by transforming both the torus parameters (G_{ij}, B_{ij}) and the oscillators as follows

$$\begin{aligned} G' + B' &= (a[G + B] + b)(c[G + B] + d)^{-1} \\ G' + B' &= (-[G + B]c^T + d^T)^{-1}([G + B]a^T - b^T) \\ G' &= (V_L^{-1})^T G V_L^{-1} = (V_R^{-1})^T G V_R^{-1} \\ \alpha_n'^{Li} &= (V_L)_j^i \alpha_n^{Lj}, \quad \alpha_n'^{Ri} = (V_R)_j^i \alpha_n^{Rj} \\ V_L &\equiv [c(G + B) + d], \quad V_R \equiv [c(-G + B) + d] \end{aligned} \quad (2.4)$$

where the second and first equations are equal by using (2.3). The third equation is obtained from the first or second by projecting on the symmetric part of both sides. So, the oscillators undergo transformations V_L, V_R that depend on the $O(c, c; Z)$ parameters as well as on the target space background fields G, B .

For our purposes we get a clearer picture by defining oscillators in a flat basis rather than the “curved” basis of G_{ij} . Thus, by introducing vielbeins $G_{ij} = e_i^a e_j^a$ we have the flat oscillators

$$\begin{aligned} \alpha_n^{La} &= e_i^a \alpha_n^{Li}, \quad \alpha_n^{Ra} = e_i^a \alpha_n^{Ri} \\ N_L &= \sum \alpha_{-n}^{La} \alpha_n^{La}, \quad N_R = \sum \alpha_{-n}^{Ra} \alpha_n^{Ra} \end{aligned} \quad (2.5)$$

whose transformation properties under $O(c, c; Z)$ amount to induced maximal compact subgroup rotations $O(c)_L \times O(c)_R$. This is seen by noting that orthogonal transformations are induced on the flat index of the

vielbein in order to maintain the transformation law for the metric

$$\begin{aligned} (e_i^a)' &= (V_L^{-1})_i^j e_j^b (T_L)_b^a \\ (e_i^a)'' &= (V_R^{-1})_i^j e_j^b (T_R)_b^a \\ T_L &\subset O(c)_L, \quad T_R \subset O(c)_R \end{aligned} \quad (2.6)$$

We emphasize that $T_L \neq T_R$ since $V_L \neq V_R$, so that the left and right moving flat oscillators $\alpha_n^{La}, \alpha_n^{Ra}$ transform under different orthogonal transformations.

Thus, for massive states the *indices* must form complete representations

$$\begin{aligned} \text{indices} &\longleftrightarrow SO(d-1) \otimes k \\ k &= O(c)_L \times O(c)_R. \end{aligned} \quad (2.7)$$

where $SO(d-1)$ is the rotation group that identifies the spin of the state in d -dimensions. In the $l=0$ sector the little group $SO(d-2)$ replaces the $SO(d-1)$ factor¹. We note that k is larger than the naive $O(c)$ that is embedded in $SO(10) \rightarrow SO(d-1) \otimes SO(c)$.

In the superstring theory of type-II, which is of interest in this paper, the above discussion does not change in the presence of fermions. The fermions are treated in the Green-Schwarz formalism. The “vacuum” state is the Clifford vacuum for the zero mode fermions, that produces all the massless states in compactified 11-dimensional supergravity. This set of states, already form U -multiplets in supergravity by definition (see also below), and therefore are consistent with the classification with $k \subset K \subset U$. Furthermore, the fermion oscillators end up in the spinor representation of the maximal compact subgroup $k = O(c)_L \times O(c)_R$. Therefore, the states that they create at higher levels l are consistent with the index structure of $O(c)_L \times O(c)_R$. The upshot is that the superstring theory has T -multiplets that are classified precisely as in (2.1) for various spins, including fermions. Later in section III, by explicit construction of the states we will identify the k -multiplets in type II string theory at each oscillator level.

III. U-MULTIPLETS AT LEVEL 0, “INDEX” AND “BASE”

In this section we study the structure of U -multiplets at level $l=0$ (including non-perturbative states) and show that they are of the proposed form (1.3).

¹Actually the type-II superstring massless states do form larger multiplets under $SO(d-1)$. This is related to the fact that there is an 11D structure in the type-II SUSY.

A. Massless states and indices

The low energy sector of the type-II string theory is described by compactified 11-dimensional supergravity. The fields correspond to the vacuum sector of the superstring in the Green-Schwarz formalism

$$\phi_{indices}^{(0)}(x^\mu) \longleftrightarrow |vac, p^\mu > \quad (3.1)$$

The indices come from the R-R vacuum in the Green-Schwarz formalism. They correspond to the short supermultiplet $2_B^7 + 2_F^7$ of 11D supersymmetry, dimensionally reduced to $R^d \otimes T^{c+1}$. This is shown explicitly in the appendix. Here one can start to re-classify the indices under U by using the original Julia-Cremmer classification [17] of the massless states, as given in the appendix. It is seen that the fermions are not in U -multiplets, but rather in K -multiplets. Furthermore, the scalars classified in the coset U/K undergo a non-linear transformation under U . So, the U -transformations that leave the supergravity Lagrangian invariant act in non-linear ways.

The pattern of U -transformations is clearer in the description of supergravity as a gauged sigma model. The scalars start out as a matrix $\exp(t \cdot \phi)$ in the adjoint representation of U . One must distinguish *global* discrete transformation under the U_{global} -group that acts on one side of the matrix of scalars, and *local* maximal compact subgroup $K_{local} \subset U'$ transformations that act on the other side of the matrix of scalars (where U' is isomorphic to U but commutes with it). The fermions are classified under K_{local} while the massless vectors and massless antisymmetric tensors are in representations of U_{global} . The gauge fields for the group K are non-propagating, so one can choose a unitary gauge and eliminate the auxiliary gauge fields through the equations of motion. The remaining physical scalars are classified in the coset U/K . In the unitary gauge, the transformations of U_{global} on the scalars have to be compensated with transformations of K_{local} in order to maintain the gauge. The fermions transform under this field dependent K_{local} whose free parameters are just the ones in U_{local} . The massless vectors and tensors continue to transform under U_{global} . Thus, we outline the $l = 0$ pattern of “indices” as follows

spin	action of U -group	representation
scalars	$U_{global} \otimes K_{local}$	coset U/K
vectors, tensors	U_{global}	dim. in appendix
spinors	K_{local}	dim. in appendix

TABLE II. U -group acts in two ways in supergravity.

This classification is the starting point for U -multiplets. It already hints that one must analyze the whole string spectrum from the point of view of both global- U and local- K transformations that commute with each other. However in the unitary gauge, which is the form in which string theory presents itself, it is hard to

keep track of the “two sides” of the matrix of scalars. Therefore, in this paper we will be less ambitious and we will explore some consequences of the *diagonal subgroup* K_{diag} that sits both in the global and local sides. It is this K_{diag} subgroup that classifies the “indices” of the fields in (1.3). We observe that the classification of all the massless fields in table II is consistent with (1.3) since all of them correspond to complete K -multiplets.

B. Massive fields in $l = 0$ multiplet

Next consider all the massive perturbative string states of the form (2.1) in the $l = 0$ sector that correspond to Kaluza-Klein and winding solutions labelled by (\vec{m}, \vec{n})

$$\phi_{indices}^{(0)}(x^\mu, \vec{m}, \vec{n}) \longleftrightarrow |vac, p^\mu, \vec{m}, \vec{n} > . \quad (3.2)$$

These have the same set of *indices* as the massless fields corresponding to the same $2_B^7 + 2_F^7$ short multiplet of 11D SUSY discussed above. These massive fields can simply be regarded as if coming from the Kaluza-Klein compactification of the supergravity fields, but extended by T-duality transformations of the base. Evidently they exist in the string theory and they form T-multiplets consistent with (1.3).

Now we discuss the non-perturbative BPS saturated soliton solutions which have non-perturbative charges z^I . The existence of some such states is proven by finding soliton solutions to the field equations of supergravity [1]- [10]. This shows that the supersymmetry algebra has non-trivial non-perturbative central charges Z that commute both with the momentum and supersymmetry generators.

The supercharges, momenta and central charges form an algebraic system which must be quantum mechanically represented in the spectrum. That is, the eigenstates of the commuting operators, including the central charges, *should* be included as labels of the states of the full theory. In our discussion of the base space below we will show algebraically that the non-perturbative z^I are at the same footing as the perturbative (\vec{m}, \vec{n}) , so the complete set of states in the $l = 0$ sector must be of the form

$$\phi_{indices}^{(0)}(x^\mu, \vec{m}, \vec{n}, z^I) \longleftrightarrow |vac, p^\mu, \vec{m}, \vec{n}, z^I > \quad (3.3)$$

where the indices again correspond to the $2_B^7 + 2_F^7$ short multiplet of dimensionally reduced 11D SUSY. For fixed values of the *base* this multiplet correspond to BPS saturated states, hence the mass is a function of the charges (\vec{m}, \vec{n}, z^I) . This provides an algebraic construction of the states that were found in the form of classical solutions. Actually, there are many more states, with definite quantum numbers that are uniquely specified by U -transformations (given below). Furthermore, it puts the perturbative and non-perturbative states on the same footing. We will give below the precise U -transformation

that mixes them. Thus, the U-multiplet is in complete agreement with the form (1.3).

A well known first example is the uncompactified theory with $(d, c) = (10, 0)$. There are no (\vec{m}, \vec{n}) , but there is one non-perturbative central charge z . The states in the $l = 0$ sector are $\phi_{indices}^{(0)}(x^\mu, z) \longleftrightarrow |vac, p^\mu, z\rangle$. The classical solutions with non-trivial z are known as BPS saturated black holes [1]. Witten [2] interpreted this set of fields as 11D supergravity fields compactified on $R^{10} \times S^1$, thus recovering an additional dimension. The U-group in this case is trivial, however this well known case serves to illustrate that our reasoning so far is consistent with previous discussions. Furthermore, it shows that there are signs of a hidden 11D structure at $l = 0$, and serves as a starting point for investigating 11D structure at higher levels l [12].

C. U-classification of the Base Space

Let us now clarify the content and transformation properties of the *base*. For the moment we concentrate on the BPS saturated black holes that are 0-brane solutions of the 11D supergravity equations, and return to the more general p -branes at the end of the paper. In that case we label the base as above

$$base = (\vec{m}, \vec{n}, z^I) \quad (3.4)$$

We will show that these quantum numbers correspond to the central charges of 11D supergravity and at the same time that they are the sources that couple to the massless vector fields of 11D supergravity listed in the appendix. Combining the two statements we will learn the U-classification of the base.

Consider the supersymmetry algebra of type-IIA in 10D. There are two supercharges of opposite chirality. Together they form the 32-component spinor of 11D. In the compactified theory the 32 components are labelled as Q_α^a where α, a label the spinor representations in the Minkowski and compactified dimensions respectively. The centrally extended superalgebra is

$$\{Q_\alpha^a, Q_\beta^b\} = \delta^{ab} \gamma_{\alpha\beta}^\mu P_\mu + 1_{\alpha\beta} Z^{ab} \quad (3.5)$$

The $1_{\alpha\beta}$ denotes either a symmetric or antisymmetric combination of Lorentz (spinor) indices into a Lorentz singlet. Then the corresponding central charges Z^{ab} are either symmetric or antisymmetric respectively. Here we will show that the central charges Z^{ab} form complete K and U multiplets and that these correspond exactly to the same multiplets that describe the massless vector fields listed in the appendix. This fact is proven by building Table-III in three steps: (i) Classify the supercharges under $SO(d-1, 1) \otimes k$, noting that $k = SO(c)_L \otimes SO(c)_R$ naturally emerges from the two chiral supercharges, (ii) Construct all possible Z^{ab} whenever a $1_{\alpha\beta}$ is allowed by the Lorentz content, and note the k -classification, (iii)

Combine the k -representations precisely to complete K representations, as listed.

d/c	$32 Q_\alpha^a$, under $SO(d-1, 1) \otimes k$	Z^{ab} under k	Z^{ab} under K
10/0	$16_L + 16_R$	1	1
9/1	$16_+ + 16_-$	$\begin{pmatrix} + & - & + \\ + & - & - \end{pmatrix}$	$\begin{pmatrix} + & - & + \\ + & - & - \end{pmatrix}$
8/2	$\begin{pmatrix} 8_L^+, (\pm)_L \\ + (8_R^-, (\pm)_R) \end{pmatrix}$	$\begin{pmatrix} + & - & + \\ + & - & - \end{pmatrix}_L$ $\begin{pmatrix} + & - & + \\ + & - & - \end{pmatrix}_R$	$3_+ + 3_-$
7/3	$(8, [(2, 0) + (0, 2)])$	$(3, 0)$ $+ (0, 3)$ $+ (2, 2)$	10
6/4	$\begin{pmatrix} 4_L, (4_{spin}, 0) \\ + (4_R, (0, 4_{spin})) \end{pmatrix}$	$(4_{spin}, 4_{spin})$	$(4, 4)$
5/5	$(4, [(4, 0) + (0, 4)])$	$(5, 0)$ $+ (0, 5)$ $+ (4, 4)$ $+ 2(0, 0)$	$27+1$
4/6	$(4_{spin}, [(4, 0) + (0, \bar{4})])$	$(6, 0)$ $+ (0, \bar{6})$ $+ (4, \bar{4})$	28_{complex}
3/7	$(2, [(8, 0) + (0, 8)])$	$(21, 0)$ $+ (7, 0)$ $+ (0, 21)$ $+ (0, 7)$ $+ (8, 8)$	120

TABLE III. Classification of central charges

Now, by comparing to the U-representations of the vectors listed in the appendix we see that both the counting and the K -representation content is the same² as the Z^{ab} . Hence, the central charges

$$Z^{ab} = (\vec{m}, \vec{n}, z^I) \quad (3.6)$$

form a complete U-multiplet.

The central charges and the sources for the massless vectors are related as follows. The Kaluza-Klein and winding charges (\vec{m}, \vec{n}) are the ‘‘perturbative charges’’ among the Z^{ab} . In the NSR formalism they couple to the massless vectors that come from the NS-NS sector. From the point of view of 10D supergravity compactified on $R^d \otimes T^c$, these massless vectors are the gravi-photon and its axial partner which come from the dimensional reduction of the 10D metric $g_{\mu\nu}$ and antisymmetric tensor $B_{\mu\nu}$

$$g_{\mu\nu} \rightarrow \vec{V}_\mu, \quad B_{\mu\nu} \rightarrow \vec{V}'_\mu, \quad (3.7)$$

The non-perturbative 0-brane solutions have charges z^I that couple to the remaining massless vector fields V_μ^I .

²In the case $(d, c) = (5, 5)$ we found one extra central charge in addition to the 27, listed as $27+1$ in Table-III. According to the appendix, it seems that there is no corresponding singlet massless vector in compactified 11D supergravity. This is curious.

These vectors come from the Ramond-Ramond sector in the NSR formalism. Thus, in string theory compactified on $R^d \otimes T^c$ the base contains altogether $N_{d,c}$ charges for 0-branes, which is the same as the number $N_{d,c}$ of massless vector fields in 11D supergravity compactified on $R^d \otimes T^{c+1}$.

The non-perturbative charges z^I are now on the same footing as the perturbative charges (\vec{m}, \vec{n}) since they both are sources in the field equations of the massless vector fields. This suggests naturally that the *base* in (1.3) transforms in the same U-duality multiplet as the vector fields themselves. This is precisely the multiplet of size $N_{d,c}$ listed in the appendix. For example for $(d, c) = (6, 4)$ the vectors fall into the 16-dimensional spinor representation of $SO(5, 5)$. Hence, the *base* also transform in the same linear representation³.

IV. U-MULTIPLETS AND STRING EXCITATIONS AT $L \geq 1$

Our approach is the following: We start with T -multiplets that can be defined perturbatively at every level l . The T -multiplet indices form k -multiplets, which are in turn required to be part of complete larger K multiplets in accordance with (1.3). This last part is the consistency requirement for the T -multiplets to reassemble into U -multiplets. The base is already a U -multiplet as discussed above, so no more discussion is needed. The remaining question is whether there is a need to add non-perturbative k -multiplets (indices) to the perturbative ones in order to find K multiplets and thus satisfy U -duality. Here is a summary of what we find, and which will be described below in several steps:

We will prove that at level $l = 1$ the perturbative k -multiplets have precisely the index structure that forms complete K -multiplets. No additional indices are needed at level $l = 1$, just like the case of level $l = 0$. This gives more credibility to our proposal in eq.(1.3). This fact is quite non-trivial in various dimensions (d, c) , and we will be able to explain it as a consequence of the underlying spacetime supersymmetry.

At levels $l \geq 2$ the requirement of complete K -multiplets predicts the existence of additional non-perturbative k -multiplets beyond those that can be created by applying oscillators on the “base”. That is, more “non-perturbative” indices with predicted properties must be added. The quantum numbers of these

states are therefore completely determined. If these additional states do not exist in the theory there is no U -duality.

A. Perturbative k -multiplets in type II string at $l \geq 1$

In order to obtain the perturbative k -multiplets explicitly we examine the known spectrum of the type-IIA superstring in 10D before compactification, and notice that there are some larger symmetry structures that help in extracting directly the k structure after compactification, as explained below.

Our analysis begins by re-examining the perturbative spectrum and extracting the indices

$$indices \longleftrightarrow (\text{Bose} \oplus \text{Fermi oscillators})^{(l)} |vac\rangle \quad (4.1)$$

This was done up to level $l = 5$ in [12]. The result for $d = 10, c = 0$ is equivalent to a collection of fields $\phi_{indices}^{(l)}(x^\mu)$ where the indices have the following structure of representations

$$indices \Rightarrow (2_B^{15} + 2_F^{15}) \times R^{(l)} \quad (4.2)$$

The factor $2_B^{15} + 2_F^{15}$ represents the action of 32 supercharges on a set of $SO(9)$ representations $R^{(l)}$ at oscillator level l , where $SO(9)$ is the spin group in 10-dimensions for massive states. The factor $R^{(l)}$ is of the form of direct products of $SO(9)$ representations coming from left/right movers

$$R^{(l)} = \left(\sum_i r_i^{(l)} \right)_L \times \left(\sum_i r_i^{(l)} \right)_R \quad (4.3)$$

such that the left-factor is identical to the right-factor, and is given by the following collection of $SO(9)_{L,R}$ representations

Level	$SO(9)_{L,R}$ representations $\left(\sum_i r_i^{(l)} \right)_{L,R}$
$l = 1$	1_B
$l = 2$	9_B
$l = 3$	$44_B + 16_F$
$l = 4$	$(9 + 36 + 156)_B + 128_F$
$l = 5$	$(1 + 36 + 44 + 84 + 231 + 450)_B$ $+ [16 + 128 + 576]_F$

TABLE IV. L/R oscillator states of 10D superstring.

This structure shows that the factor $R^{(l)}$ is really classified by the larger group

$$SO(9)_L \otimes SO(9)_R. \quad (4.4)$$

Furthermore, the supercharge factor $2_B^{15} + 2_F^{15}$ has an even larger classification group

$$SO(32) \quad (4.5)$$

³In the manifestly spacetime supersymmetric Green-Schwarz formalism of the type-II string, all massless fields, including the massless vector fields, come from the R-R vacuum sector that is described by only the fermionic zero modes. In this sense the discussion so far has not involved any oscillators, so that it is appropriate to use the term “base” for all the quantum numbers included above.

with $2_B^{15} + 2_F^{15}$ corresponding to the two spinor representations. The diagonal $SO(9)$ subgroup of all these factors is the familiar rotation group in the Lorentz group $SO(9, 1)$.

When the the string theory is compactified to $R^d \otimes T^c$, with $c + d = 10$, the set of indices in (4.2) needs to be re-classified by

$$SO(d-1) \otimes [SO(c)_L \otimes SO(c)_R] \quad (4.6)$$

in order to identify the perturbative k -multiplets and their spins. Note that this is a larger group than what is contained in the rotation group alone $SO(9) \supset SO(d-1) \otimes SO(c)$. Therefore, the larger symmetry structures that we identified above are needed to obtain the k -multiplets. It is then evident that the $R^{(l)}$ factor, described by the representations in Table-IV, is easily reduced in the form

$$SO(9)_{L,R} \Rightarrow SO(d-1)_{L,R} \otimes SO(c)_{L,R}. \quad (4.7)$$

For example, for $d = 6$, $c = 4$

$$\begin{aligned} l = 1 : & \quad 1 \rightarrow 1 \\ l = 2 : & \quad 9_{L,R} \rightarrow (5, 1)_{L,R} + (1, 4)_{L,R}, \quad etc. \end{aligned} \quad (4.8)$$

After this step the $SO(d-1)_L \times SO(d-1)_R$ is reduced to the diagonal rotation group $SO(d-1)$ in order to give the final classification of states. Naturally the outcome is the desired classification under $SO(d-1) \otimes [SO(c)_L \otimes SO(c)_R]$.

Similarly, the $2_B^{15} + 2_F^{15}$ supercharge factor may be reduced to the representations of

$$SO(d-1) \otimes [SO(c)_L \otimes SO(c)_R] \quad (4.9)$$

since we have already seen in Table-II that the 32 supercharges are already classified under this group. Then the reclassification of the factor $2_B^{15} + 2_F^{15}$ follows by taking the product of the supercharges. Equivalently, specifying how to decompose the 32-dimensional vector of $SO(32)$, provides the instructions for decomposing the 2^{15} dimensional representations as well.

Therefore all the perturbative indices in (4.2) do form k -multiplets that we can identify explicitly thanks to the larger structures $SO(32)$ and $SO(9)_L \otimes SO(9)_L$. Combining this result with the base (\vec{m}, \vec{n}) proves that we have explicit control of the T -multiplets $\phi_{indices}^{(l)}(x^\mu, \vec{m}, \vec{n})$ up to level $l = 5$. We will next construct U -multiplets by starting with these T -multiplets.

B. U -multiplets at $l = 1$

In the discussion above we used the explicit $SO(d-1) \otimes k$ classification of the 32 supercharges of Table-III. Here we go one step further and give in Table-V the re-classification of the 32 supercharges under the group $[SO(d-1) \otimes K]$.

$\frac{d}{c} \rightarrow \frac{d-1}{c+1}$	32 SUPERCHARGES Q_α^a , under $SO(d-1) \otimes K$
10/0 \rightarrow 9/1	16 + 16
9/1 \rightarrow 8/2	$(8_+, \pm) + (8_-, \pm)$
8/2 \rightarrow 7/3	$(8, 2, \pm)$
7/3 \rightarrow 6/4	$(4, 4) + (4^*, 4)$
6/4 \rightarrow 5/5	$(4, 4, 0) + (4, 0, 4)$
5/5 \rightarrow 4/6	$((2, 0), 8) + ((0, 2), 8)$
4/6 \rightarrow 3/7	$(2, 8) + (2, 8^*)$
3/7 \rightarrow 2/8	$(\pm, 16)$

TABLE V. Classification of 32 Q_α^a exhibits K

Table-V was constructed in several steps: (i) The vector of $SO(32)$ is identified with the 32-dimensional spinor of 11-dimensions. (ii) This spinor is decomposed into the two 16-dimensional spinor representations of $SO(10)$, where $SO(10)$ is the rotation group in 11-dimensions. (iii) The $SO(10)$ spinor representations are decomposed into the products of spinors of $SO(d-1) \times SO(c+1)$ where $SO(d-1)$ is the rotation group in d -dimensional Minkowski space, (iv) Then we find that the resulting spinor representations of $SO(c+1)$ come together with the correct numbers to fit into the complete K -representations given above.

The subgroup K turns out to be the largest subgroup in $SO(32)$ that remains after identifying the spin group $SO(d-1)$ as described above. So, K emerges in an interesting way as this largest factor. Each K -multiplet then contains several k -multiplets of the supercharges automatically. For example, for $d = 6$, $c = 4$ there are two supercharges classified as $(4, 0) + (0, 4)$ under the $K = SO(5) \times SO(5) = Sp(4) \times Sp(4)$. They could be further re-classified under $k = SO(4) \times SO(4)$. Recall that our goal is to reassemble all representations into K -multiplets. With this table we have demonstrated that this is always the case for any representation of $SO(32)$ that is a product of the 32 supercharges. Hence the supercharge factor $2_B^{15} + 2_F^{15}$ admits a reclassification under K for all compactifications (d, c) .

There remains to discuss the K -reclassification of the $R^{(l)}$ factor in the perturbative indices in (4.2) separately for every level l . According to table III, at level $l = 1$ we have just a singlet

$$l = 1 : \quad R^{(1)} = 1.$$

Obviously, this is easily re-classified also as a singlet under K . Therefore, for all compactifications (d, c) we have now demonstrated that level $l = 1$ states do have indices that correspond to complete K -multiplets. Hence at level $l = 1$ we have U -multiplets of the form (1.3) without needing any additional non-perturbative indices. The only non-perturbative aspects at level $l = 1$ come through the non-perturbative charges z^I at the base.

C. U-multiplets at $l \geq 2$

Next we analyze levels $l \geq 2$. We know that the supercharge factor $2_B^{15} + 2_F^{15}$ works, so we ignore it, and concentrate on the $R^{(l)}$ factor. It is not straightforward to carry out the analysis simultaneously for every (d, c) . Therefore, we need to do it one case at a time. It is very easy to analyze the case $(d, c) = (6, 4)$ so we present it here as an illustration. In this case the spin group is $SO(5)$ and there are 4 internal dimensions. The duality groups are

$$\begin{aligned} U &= SO(5, 5), \quad K = SO(5) \otimes SO(5) \\ T &= SO(4, 4), \quad k = SO(4)_L \otimes SO(4)_R \\ R^{(2)} &= (5_{space} + 4_{L,int}) \otimes (5_{space} + 4_{R,int}), \quad (4.10) \\ R^{(3)} &= etc. \end{aligned}$$

where the indices $R^{(2)} = 9_L \otimes 9_R$ have been reclassified according to their space and internal components. It is clear from this form that the $k = SO(4)_L \otimes SO(4)_R$ structure follows directly from the left/right internal components. This identifies the specific k -multiplets in $R^{(2)}$ for various spins. Can these $SO(4)_L \otimes SO(4)_R$ multiplets be reassembled into $SO(5) \otimes SO(5)$ multiplets for every spin? The answer is obviously no! This means that some non-perturbative states are missing. The structure of the indices that are missing corresponds precisely to increasing the

$$(4_{int})_{L,R} \rightarrow (5_{int})_{L,R}. \quad (4.11)$$

This is equivalent to requiring non-perturbative states that correspond to increasing the size of the $l = 2$ multiplet in the form

$$R^{(2)} = 9_L \otimes 9_R \rightarrow 10_L \otimes 10_R. \quad (4.12)$$

This result was found in [12] by assuming the presence of hidden 11-dimensional structure in the non-perturbative type-IIA superstring theory in 10D. In ref. [12] a justification for (4.12) could not be given. However, in the present analysis U -duality demands (4.11) and therefore justifies (4.12).

For $(d, c) = (10, 0), (9, 1), (8, 2), (6, 4)$ the analysis for $l = 2, 3, 4, 5$ produces exactly the same conclusion as the 10D analysis, in that U -duality demands that the $SO(9)_L \otimes SO(9)_R$ multiplets $R^{(l)}$ should be completed to $SO(10)_L \otimes SO(10)_R$ multiplets. The minimal completion is sufficient in this case. The necessary minimal completion was given and discussed in [12], where the possibility of additional non-perturbative complete $SO(10)$ multiplets is also discussed. For all higher levels $l = 2, 3, 4, 5$ there is a clear remarkable pattern of the *minimal* missing non-perturbative states. Their quantum numbers (for both bosons and fermions) coincide systematically with the sum of all lower lying perturbative states listed in Table-IV. This observation systematically gives all the *minimal* states required by U -duality at all levels.

Hence, in these compactifications U -duality is consistent with a hidden 11D structure.

Next consider the example $(d, c) = (7, 3)$. Now we have a spin group $SO(6) = SU(4)$

$$\begin{aligned} U &= SL(5), \quad K = SO(5) = Sp(4) \quad (4.13) \\ T &= SO(3, 3), \quad k = SO(3)_L \otimes SO(3)_R = SO(4) \\ R^{(2)} &= (6_{space} + 3_{L,int}) \otimes (6_{space} + 3_{R,int}), \end{aligned}$$

The $k = SO(3) \otimes SO(3)$ structure is obvious from the internal dimensions $(3_{int})_{L,R}$. Can these be put together into complete $K = SO(5)$ multiplets? The answer is no, but this is not surprising since we have already seen that the $l = 2$ perturbative states are insufficient. Can we add sufficient number of non-perturbative states to make complete K -multiplets? The answer is, of course, yes, but the needed states go beyond the minimal extension (4.12). There is a minimal number of states that we can add to obtain complete U -multiplets, but these do not agree with the minimal number that make $SO(10)$ multiplets.

These examples show that we can always find a minimal set of non-perturbative states to make complete U -multiplets in accordance with eq.(1.3). However, the systematics of the missing states is not always in accordance with the systematics of *minimal* number of 11D multiplets.

V. IS THERE HIDDEN 11D STRUCTURE?

In the previous section we saw that for

$$(d, c) = (10, 0), (9, 1), (8, 2), (6, 4)$$

U -duality is consistent with the presence of hidden 11-dimensional structure at *all string levels*. However for the other values of

$$(d, c) = (7, 3), (5, 5), (4, 6), (3, 7)$$

we found that the index structure required by the conjectured U -duality is different than the *minimal number* of 11-dimensional supersymmetry multiplets at excited levels $l \geq 2$. Does this mean we have to give up the idea that there is a hidden 11D structure? In view of recent arguments in favor of 11D [2], [11]- [14], the idea of 11D seems to become more compelling. If both U -duality and 11D are true then together they must give powerful restrictions on the structure of the theory from two different non-perturbative perspectives. The first test is to find a resolution for the spectrum when the two requirements seem to conflict with each other, as above. The only possible solution is that there exists a collection of states that is bigger than the minimal set required by either K or $SO(10)$. The property of this collection must be that it can be reclassified either as K -multiplets or $SO(10)$ multiplets. When classified as $SO(10)$ states the

K structure may be obscured or vice versa. This is a testable hypothesis.

Thus, let's reconsider the case of $(d, c) = (7, 3)$ at level $l = 2$. The perturbative states are summarized by the 81 indices in the following $SO(6) \otimes [SO(3)_L \otimes SO(3)_L]$ representations (besides the $2_B^{15} + 2_F^{15}$ factor)

$$\begin{aligned} R^{(2)} &= 9_L \otimes 9_R \rightarrow (6_{space} + 3_{L,int}) \otimes (6_{space} + 3_{R,int}) \\ &= (1 + 15 + 20, [(0, 0)]) + \\ &\quad (6, [(3, 0) + (0, 3)]) + (0, [(3, 3)]) \end{aligned} \quad (5.1)$$

We are seeking $SO(6) \otimes SO(5)$ structures that are compatible with a decomposition of $SO(10)$ multiplets. We have found 94 additional non-perturbative states that combine together with the 81 perturbative ones to give 175 states with the desired properties:

$$\begin{aligned} SO(10) : 1 + 3 \times 10 + 2 \times 45 + 54 &= 175 \quad (5.2) \\ SO(6) \otimes SO(5) : (0, 0) + 5 \times (6, 0) + 2 \times (15, 0) + (20, 0) \\ &\quad + (6, 10) + 2 \times (0, 10) + (0, 14) \end{aligned}$$

To see that these match each other, and also include the 81 perturbative states, we need to decompose them according to the following schemes

$$SO(10) \rightarrow SO(6) \otimes SO(4)_1 \rightarrow SO(6) \otimes SO(3) \quad (5.3)$$

$$SO(6) \otimes SO(5) \rightarrow SO(6) \otimes SO(4)_2 \rightarrow SO(6) \otimes SO(3)$$

where $SO(4)_2 = SO(3)_L \otimes SO(3)_L = k$ is the subgroup of $T = O(3, 3)$, whereas the $SO(4)_1$ is different since it does not contain the left/right information. However, the $SO(3)$ subgroup is common to both.

The minimal $SO(10)$ multiplets that came from $(9 + 1)_L \otimes (9 + 1)_R$ are $1 + 45 + 54$. These contain perturbative and non-perturbative states. The additional ones $3 \times 10 + 45$ are purely non-perturbative. For the cases $(d, c) = (10, 0), (9, 1), (8, 2), (6, 4)$ the additional ones were not required by the minimal completion. However, presumably these additional multiplets are present also in those cases if there is a common 11D structure behind the whole theory.

In this example we have shown that it is possible for both U-multiplets and 11D multiplets to coexist even though they appeared to be at conflict at first. It is quite difficult to carry out a similar analysis for other levels l and other cases (d, c) . At this stage we have not understood enough of the theory to know whether a similar conclusion is true more generally. Therefore the issue remains open until a better approach is found to answer the question: do U-duality and 11D structure coexist? Hopefully the answer is yes, otherwise we have to decide which one is right.

VI. SUMMARY AND COMMENTS

In this work we have investigated the compactified type-IIA superstring for various values of (d, c) . We

have presented an algebraic approach which encompasses both the perturbative as well as the non-perturbative states by putting them into U-multiplets. We have reasoned that the multiplet is labelled by two spaces, “index” space and “base” space. Both spaces are extensions of similar spaces that label the perturbative T-duality multiplets. In our scheme U-transformations, much like T-transformations, do not mix perturbative states from different levels (note the definition of level for non-perturbative states, as given in the introduction). Our whole analysis critically depends on this structure which allows us to investigate the U-multiplets level by level. The paper presented evidence that this scheme is consistent (to the extent of our investigation).

We have found that at levels $l = 0, 1$ the existing index structure for perturbative states is all that is needed to define complete U-multiplets in the form $\Phi_{indices}^{(l)}(x^\mu, base)$ for all values of (d, c) , and that this result directly follows from the simplest short and long multiplet structure of 11D space-time supersymmetry.

At levels $l = 0, 1$ all non-perturbative aspects appear in the $base = (\vec{m}, \vec{n}, z^I)$. The base quantum numbers are the central charges of the 11D SUSY algebra and these correspond to the 0-brane sources that couple to the massless vector particles in supergravity. U acts as a linear transformation on the base in a representation that is identical to the one applied to the massless vector fields in compactified 11D supergravity. Furthermore the indices correspond to complete representations of K and they mix with a transformation induced by U .

Hence, for $l = 0, 1$ both index space and base space of U-multiplets have firm connections to 11D.

To have U-duality at higher levels $l \geq 2$ additional non-perturbative states are needed to complete the index structure. If these additional states are absent in the theory there is no U-duality in the full theory. We would then have to interpret the successful result at $l = 0, 1$ as a pure accident. This is not impossible since the $l = 0, 1$ cases are fully explained by SUSY. On the other hand, assuming that U-duality is true for $l \geq 2$, our approach provides an algebraic tool for identifying the non-perturbative states at every level once the perturbative states are listed.

There seems to be a non-perturbative 11D structure lurking behind the theory. In view of the existence of a classical membrane theory with some promise of its consistency at the quantum level, searching for hidden 11 dimensional structure is an interesting challenge. It is not necessary for 11D to be present in the 10D theory, but there is mounting evidence for it, including the work we presented here. We have seen that U-duality is distinct from this 11D structure, although in some cases they appeared to imply each other. We have found cases where there is a clash between the two if one is restricted to a minimal set of non-perturbative states. We have shown at least in one example that the conflicts may be resolved by adding more non-perturbative states. But

nevertheless this example clearly shows that 11D and U-duality are quite distinct from each other. If they are both true their combined effect is quite restrictive on the non-perturbative structure of the theory. Whether the conflict can be resolved generally is a major question raised by our work.

Other places where our ideas could be tested is in the proposed dualities between the Heterotic string on $R^6 \otimes T^4$ and the type-IIA string on $R^6 \otimes K_3$, as well as other similar cases involving heterotic, type-I or type-II theories. The perturbative plus non-perturbative spectrum of these theories should match each other. By using the perturbative T-multiplets of either theory as a starting point and then requiring U-multiplets at each level one should find the same full spectrum from either side.

In this paper we have not discussed multiplets with p -branes that also enter the picture [19]- [24]. However, we propose to include them algebraically as follows. Since p -branes are sources for $p+1$ forms we can draw a parallel between the central charges for p -branes in the SUSY algebra and the $p+1$ forms in compactified supergravity. By analogy to the $p=0$ case which we have discussed, we expect that the p -brane charges are classified in the same K or U multiplets corresponding to the $p+1$ forms. Carrying the analogy further we expect the base to include all the p -brane charges

$$base = (0\text{-brane charges}, \dots, p\text{-brane charges}, \dots) \quad (6.1)$$

Thus, we propose that the base consists of a U-multiplet for each p -brane. It will be interesting to study p -branes and further explore this possibility.

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VIII. APPENDIX

In this appendix we review the massless sector of the 10D type-IIA string that coincides with the fields of 11D supergravity. We need to understand the “index” structure of these fields as this will be the basis for the U -multiplet and K -multiplet structure at level $l=0$. Since our aim is to find the largest multiplet structure we will use 11-dimensional classifications. The massless multiplets are the 11-dimensional graviton, 3-index antisymmetric tensor, and gravitino. In our notation these are the $l=0$ fields

$$\phi_{indices}^{(0)} : g_{MN}, A_{MNP}, \psi_{M\bar{\alpha}}. \quad (8.1)$$

In the lightcone gauge, the physical degrees of freedom of the string are classified by $SO(8)$ which is the little group for massless 10-dimensional states. However, it is possible to regroup the $SO(8)$ representations into $SO(9)$, which is the little group for massless states in 11-dimensions, by taking $M, N = 1, \dots, 9$, while $\bar{\alpha}$ is the 16-dimensional $SO(9)$ spinor index. This well known fact is a first indication of a hidden extra dimension.

If the string theory is toroidally compactified to $R^d \times T^c$, with $d+c=10$, then the “base” acquires additional quantum numbers that correspond to Kaluza-Klein, winding, and central charge quantum numbers, while the “indices” must now be split into space and internal parts. In the lightcone notation this corresponds to decomposing the $SO(9)$ representations above to $SO(d-2) \times SO(11-d)$ in order to obtain the spin and internal symmetry content of the fields. The indices split as follows

$$\begin{aligned} M &\rightarrow m \oplus i \quad \bar{\alpha} \rightarrow \alpha a \\ m &= 1, 2, \dots, d-2 \quad \leftrightarrow SO(d-2) \\ i &= 1, 2, \dots, c+1 \quad \leftrightarrow SO(c+1) = SO(11-d) \\ \alpha &= \text{Spinor of } SO(d-2) \\ a &= \text{Spinor of } SO(11-d) \end{aligned} \quad (8.2)$$

Note that c has been augmented by “1” due to the hidden compactified 11th dimension. Then the fields are decomposed as follows

$$\begin{aligned} g_{MN} &\rightarrow g_{mn} \oplus V_m^i \oplus S^{(ij)} \\ A_{MNP} &\rightarrow A_{mnp} \oplus B_{mn}^i \oplus V_m^{[ij]} \oplus S^{[ijk]} \\ \psi_{M\bar{\alpha}} &\rightarrow \psi_{m\alpha}^a \oplus \psi_{\alpha}^{ia} \end{aligned} \quad (8.3)$$

where we have written the space indices as subscripts and internal indices as superscripts. From the point of view of spins there are scalars, vectors, tensors, and a variety of spinors. We can classify them according to $SO(d-2) \times SO(c+1)$ since this can be read-off directly from the indices above. We can count them, and obtain their total numbers to see in which representations of U or K they would fit, as explained in table II. The counting has to take into account that in some dimensions a tensor may be dual to a vector or scalar, etc. After this is taken into account we obtain the classifications given in the tables below for various values of (d, c) .

$\frac{d}{c} \rightarrow \frac{d-2}{c+1}$	$SO(d-2)$ SCALARS $S^{(ij)} \oplus S^{[ijk]} \oplus S^{duals}$
9/1 \rightarrow 7/2	$\frac{2.3}{2} + 0 + 0 = 3$
8/2 \rightarrow 6/3	$\frac{3.4}{2} + \frac{3.2.1}{1.2.3} + 0 = 7$
7/3 \rightarrow 5/4	$\frac{4.5}{2} + \frac{4.3.2}{1.2.3} + 0 = 14$
6/4 \rightarrow 4/5	$\frac{5.6}{2} + \frac{5.4.3}{1.2.3} + 0 = 25$
5/5 \rightarrow 3/6	$\frac{6.7}{2} + \frac{6.5.4}{1.2.3} + 1 \left(\tilde{A}_{mnp} \right) = 42$
4/6 \rightarrow 2/7	$\frac{7.8}{2} + \frac{7.6.5}{1.2.3} + 7 \left(\tilde{B}_{mn}^i \right) = 70$
3/7 \rightarrow 1/8	$\frac{8.9}{2} + \frac{8.7.6}{1.2.3} + (8+28) \left(\tilde{V}_m^i + V_m^{[ij]} \right) = 128$

TABLE V. The number of scalars is $\dim(U/K)$.

$\frac{d}{c} \rightarrow \frac{d-2}{c+1}$	$SO(d-2)$ VECTORS $V_m^i + V_m^{[ij]} + V_m^{duals}$
9/1 \rightarrow 7/2	2 + 1 + 0 = 3
8/2 \rightarrow 6/3	3 + $\frac{3 \cdot 2}{2} + 0 = 6 \rightarrow (3, 2)$
7/3 \rightarrow 5/4	4 + $\frac{4 \cdot 3}{2} + 0 = 10$
6/4 \rightarrow 4/5	5 + $\frac{5 \cdot 4}{2} + 1$ (\tilde{A}_{mnp}) = 16
5/5 \rightarrow 3/6	6 + $\frac{6 \cdot 5}{2} + 6$ (\tilde{B}_{mn}^i) = 27
4/6 \rightarrow 2/7	7 + $\frac{7 \cdot 6}{2} + 0 = 28 = 56$ (self dual)
3/7 \rightarrow 1/8	0, (dual to scalars $V^i + V^{ij}$)

TABLE VI. Dimensions of U_{global} multiplets.

$\frac{d}{c} \rightarrow \frac{d-2}{c+1}$	$SO(d-2)$ TENSORS $B_{mn}^i + B_{mn}^{duals}$
9/1 \rightarrow 7/2	2 + 0 = 2
8/2 \rightarrow 6/3	3 + 0 = 3
7/3 \rightarrow 5/4	4 + 1 (\tilde{A}_{mnp}) = 5
6/4 \rightarrow 4/5	5 + 0 = 5 = 10 self dual
5/5 \rightarrow 3/6	0, dual to vector B_m^i
4/6 \rightarrow 2/7	0, dual to scalar B^i
3/7 \rightarrow 1/8	0

TABLE VII. Dimensions of U multiplets.

$\frac{d}{c} \rightarrow \frac{d-2}{c+1}$	GRAVITINOS $\psi_{m\alpha}^a$ under $SO(d-2) \otimes K$
10/0 \rightarrow 8/1	56 ₊ + 56 ₋
9/1 \rightarrow 7/2	(48, \pm)
8/2 \rightarrow 6/3	(20, 2, +) + (20*, 2, -)
7/3 \rightarrow 5/4	(16, 4)
6/4 \rightarrow 4/5	((2, 3), (4, 0)) + ((3, 2), (0, 4))
5/5 \rightarrow 3/6	(4, 8)
4/6 \rightarrow 2/7	(-, 8) + (+, 8*)
3/7 \rightarrow 1/8	0, dual to fermion ψ_α^a

TABLE VIII. Dimensions of K multiplets.

$\frac{d}{c} \rightarrow \frac{d-2}{c+1}$	FERMIONS ψ_α^{ia} , under $SO(d-2) \otimes K$
10/0 \rightarrow 8/1	8 ₊ + 8 ₋
9/1 \rightarrow 7/2	(8, \pm) + (8, \pm)
8/2 \rightarrow 6/3	(4, 4, +) + (4*, 4, -) + (4, 2, +) + (4, 2, -)
7/3 \rightarrow 5/4	(4, 16)
6/4 \rightarrow 4/5	((2, 0), (16, 0)) + ((2, 0), (4, 0)) + ((0, 2), (0, 16)) + ((0, 2), (0, 4))
5/5 \rightarrow 3/6	(2, 48)
4/6 \rightarrow 2/7	(+, 56) \oplus (-, 56*)
3/7 \rightarrow 1/8	128 ($\psi_\alpha^{ia} + \psi_\alpha^a$)

TABLE IX. Dimension of K multiplets.

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